A Moving Mesh Algorithm for Electromagnetic Devices Optimization Using Finite Element Method

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Abstract —An efficient Moving mesh algorithm with fast solver is proposed for remeshing the parameterized computation domain so as to embed finite element methods into optimization algorithms for optimizing the shapes of electromagnetic devices. The proposed method has the merits of conserving the original mesh structure with minimal mesh deformation. The developed algorithm has been applied to TEAM workshop problem No. 25. The reported results are used to showcase the efficiency and validity of the algorithm.

I. Introduction

Optimization of the shapes of electromagnetic devices has become an important issue in product design. In the optimization procedure, finite element method (FEM) is usually used to compute the objective function [1]. During the optimization process, the shapes of electromagnetic devices are inevitably changed; therefore the mesh of the devices needs to be regenerated for each FEM computation. It is also well known that data transfer from the variables of the optimization method to new meshes in FEM computation is always a challenge. For example, if the geometry of the computation domain is built and the mesh on that is generated by commercial software which is separated from the FEM program, it is always difficult to rebuild the geometry and regenerate the meshes automatically. Also, the regeneration of mesh is time consuming, especially in 3 dimensional (3D) problems; and the newly generated mesh generally has no relationship with the previous one, making the previous FEM solution not directly available for the new FEM computation in solving nonlinear problem.

Mesh smoothing algorithms have been extensively exploited to improve the mesh quality and, among others, Laplacian smoothing is one of the most popular methods [2]. Lately a related type of smoothing, namely Winslow smoothing, is proposed to alleviate or guard against mesh folding in the process of mesh smoothing [3]. Optimization-based smoothing methods are used to guarantee an improvement in the mesh quality by minimizing a particular mesh quality metric [4].

In this paper, mesh smoothing methods are introduced to move the meshes in the optimization procedure in which the position of the nodes on the boundary are changed. An efficient mesh moving algorithm with fast solver is proposed for moving the meshes on the parameterized computation domain while embedding FEM into the optimization method in order to optimize the shapes of electromagnetic devices.

In the proposed algorithm, the mesh of the initial shapes of the electromagnetic devices is generated and then with a change in the shapes of the device, the initial mesh will be moved adaptively to the new design. The proposed algorithm can be applied to remesh complex two-dimensional (2D) and three-dimensional (3D) geometrical shapes and save considerable time when regenerating the meshes. The new moving mesh algorithm has the merits of conserving the original structure of the mesh with minimal mesh deformation, as the mesh smoothing methods can smooth out the incremental positions of all the mesh points in the computation domain. Another merit of the proposed method is that when solving time stepping equation or nonlinear equation, the result of the previous FEM solution can serve as a good initial solution for subsequent FEM computation, as the increment of each point is small and each point is in the same material after reconstruction of the mesh. Furthermore, in the proposed algorithm, the mesh information is stored and modified in memory, which avoids the need to retrieve the mesh information from the disk at each optimization step.

II. Moving Mesh Algorithm

When optimizing several parameters of an electromagnetic device, the shape of the device is changed at each optimization step. A novel moving mesh method is proposed to obtain the new mesh according to the new shape of the device, instead of rebuilding the geometry and regenerating the mesh.

In the proposed moving mesh algorithm, an initial shape of the electromagnetic device is set up according to the range of the optimization parameters. Usually the parameters of the initial shape are the average value of its upper and lower bounds. An initial mesh is generated on the initial shape of the device. At each optimization step, changes in the parameters of the device can be considered as a change in the boundary of the device. According to the changes in boundary of the computation domain, the mesh can be moved based on the initial mesh. In this paper, the mesh moving algorithm is described in 2D, but it can be extended to 3D problems in a straightforward manner.

A. Laplacian Smoothing

The basic idea of the moving mesh algorithm based on Laplacian smoothing can be illustrated by a simple example in Fig. 1. The initial mesh of a fixed initial shape of the electromagnetic device is shown in Fig. 1 (a). The shape of the boundary Γ_1 is parameterized and it needs to be optimized while the shape of Γ_2 remains unchanged.

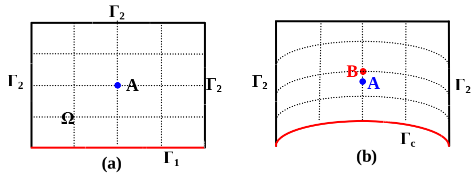

Fig. 1. (a) Mesh on the original geometry. (b) Mesh on the geometry of the changed shape.

When the shape of Γ_1 in Fig. 1(a) needs to be changed to Γ_c as shown in Fig. 1(b), the coordinates of some interior mesh points in Fig. 1(a) should be adjusted accordingly. The incremental coordinates of each interior points $\boldsymbol{u}=(\Delta x, \Delta y)$ are variables and satisfy the following Laplace's equation, as the Laplace's equation can smooth out the incremental coordinates of the mesh points inside the computation domain:

$$-\frac{\partial}{\partial x}\left(\frac{\partial \boldsymbol{u}}{\partial x}\right)-\frac{\partial}{\partial y}\left(\frac{\partial \boldsymbol{u}}{\partial y}\right)=0, \text{ in } \Omega, \quad (1)$$

$$\boldsymbol{u}=0, \quad \text{on } \Gamma_2,$$

$$\boldsymbol{u}=\boldsymbol{u}_0(b)-\boldsymbol{u}_0(a), \text{ on } \Gamma_1.$$

where; $\boldsymbol{u}_0(b)$, $\boldsymbol{u}_0(a)$ are the coordinates of the point on Γ_1 and Γ_c, respectively.

The mesh point coordinates in Fig. 1(b) can be obtained by the sum of the solution of equation (1) and their original coordinates in Fig. 1(a).

B. *Weighted Laplacian Smoothing*

Laplacian smoothing method is easy to implement and efficient. However the method is not guaranteed to work sometimes when inverting mesh elements. A weighted Laplacian smoothing method based on optimization method which is more resilient to mesh folding is introduced in [4]. The weighted Laplacian smoothing method is used with the aim to guarantee an improvement in the mesh quality by minimizing a particular mesh quality metric. Compared to other smoothing method, their main drawback is their computational expense in evaluating the weight on each edge in the mesh.

III. Numerical Experiment

The proposed method is applied to TEAM workshop problem No. 25 [5]. The goal of this problem is to optimize the shape of a die mold to obtain the best performance of permanent magnets. Fig. 2(a) is the model of the die mold with the electromagnet for the orientation of the magnetic axis of the magnetic powder. The die mold is described by an internal circle of radius R_1 and by an external ellipse represented by L_2, L_3 and L_4, as shown in Fig. 2(b). The objective function W for the optimization problem is given by:

$$W=\sum_{i=1}^{n}\left[\left(B_{xp_i}-B_{xo_i}\right)^2+\left(B_{yp_i}-B_{yo_i}\right)^2\right]$$

where n is the number of specified points (n=10); B_{xpi} and B_{ypi} are computed values along the line e-f; B_{xi} and B_{yo} are specified as $B_{xo}=1.5\cos(\theta)$, $B_{yo}=1.5\sin(\theta)$ (T).

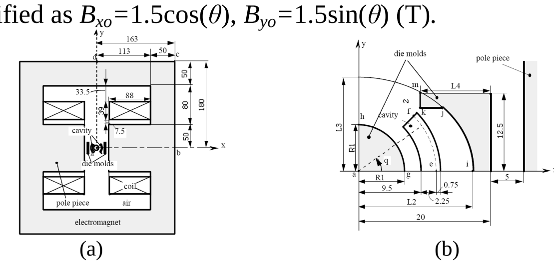

Fig. 2. (a) Die mold with electromagnet. (b) Optimization parameter of the die mold.

The mesh on the optimal shape of die mold with the parameters R_1= 8.94, L_2 = 17.85, L_3 = 15.28, L_4 = 16.68 is shown in Fig. 3(b). Two cases of extreme deformation of the shape of the die mold are studied and there is no folding in the mesh, as shown in Fig. 3 (c) and (d), respectively.

To do 100 times of remeshing, it takes about 21 seconds using moving mesh algorithm while 55 seconds are needed to regenerate mesh with commercial software. In other words, a significant of computing can be saved using the proposed technique.

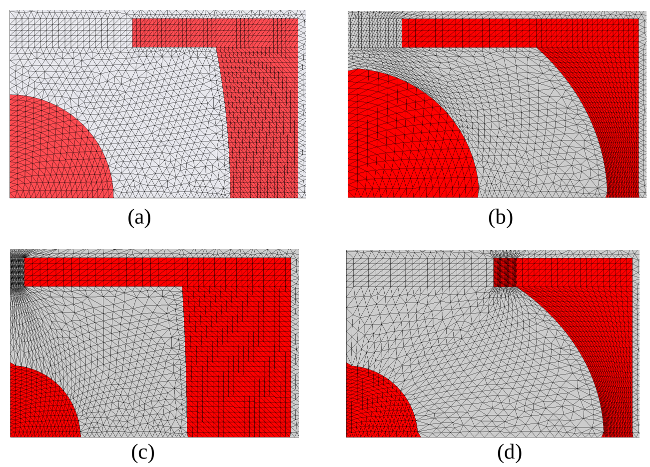

Fig. 3. Mesh on (a) initial shape of die mold; (b) optimal shape of the problem; (c) first case of extreme deformation of the die mold (d) second case of extreme deformation of the die mold

IV. References

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